

Analysis of Angular Correlations in $\gamma - \gamma$ Cascades Using Coincidence Detection and Monte Carlo Simulation

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Abstract.

Coincidence counting techniques are widely used to measure correlations between γ -ray emissions from nuclear decays. In this work, a computational framework is developed to support future angular-correlation measurements using a segmented multi-detector system. A common geometry engine provides a consistent detector description across deterministic modelling, Monte Carlo simulation, and experimental data analysis, while a custom FLUKA source routine implements angular correlations through sampling of the cumulative distribution function derived from the theoretical angular correlation function, $W(\theta)$. Agreement between deterministic and Monte Carlo models validates the computational toolchain and demonstrates the ability to correct for the expected angular response of the detector system. The validated toolchain provides the foundation for commissioning the physical detector system, interpreting future angular-correlation measurements, and extending the methodology to more complex decay schemes and detector geometries.

1 Introduction

During radioisotope nuclear decay cascades, gamma rays emitted in rapid succession exhibit an angular correlation between their relative emission directions. The theoretically expected angular correlation describes the opening angle between photon pairs as an expansion of Legendre Polynomial terms [1], which can be compared against experimental results for cascade pairs measured in time coincidence. For any single emitted photon there is no preferred nuclear alignment in the laboratory reference frame and the emission is isotropic. When γ -ray pairs are correlated by the cascade process (where successive emissions occur on a timescale shorter than the nuclear relaxation time) the relative emission detections exhibit the correlation. Practically, detection of one of the pair is used as a trigger to define the initial direction, and the intensity of the other is measured as a function of angle.

The event geometry is illustrated in Fig. 1. (left), characterised by the opening angle between the two photon directions (θ_{uv}), with the singles distributions characterised by the azimuthal angle of each direction relative to a fixed laboratory axis (ϕ_u, ϕ_v). The angular correlation is observable for detection of the photon pairs where the direction of one photon is projected along a common axis (left to right in Fig. 1 right) and the plane of emission for the second photon is rotated into the plane of the figure. The intensity of detected pairs is measured as a function of angle as in the polar plot shown in Fig. 1 right, with a uniform circle resulting from no angular correlation (i.e. isotropic emission of both photons). The detection probability of cascade pairs is modulated by the angular correlation function, $W(\theta)$, with θ the angle between γ -ray pairs, and conventionally normalised such that $W(90^\circ) = 1$. The expected angular correlation is written in terms of a Legendre polynomial expansion with coefficients a_2 and a_4 as:

$$W(\theta) = 1 + a_2 \cos^2 \theta + a_4 \cos^4 \theta \quad [1]$$

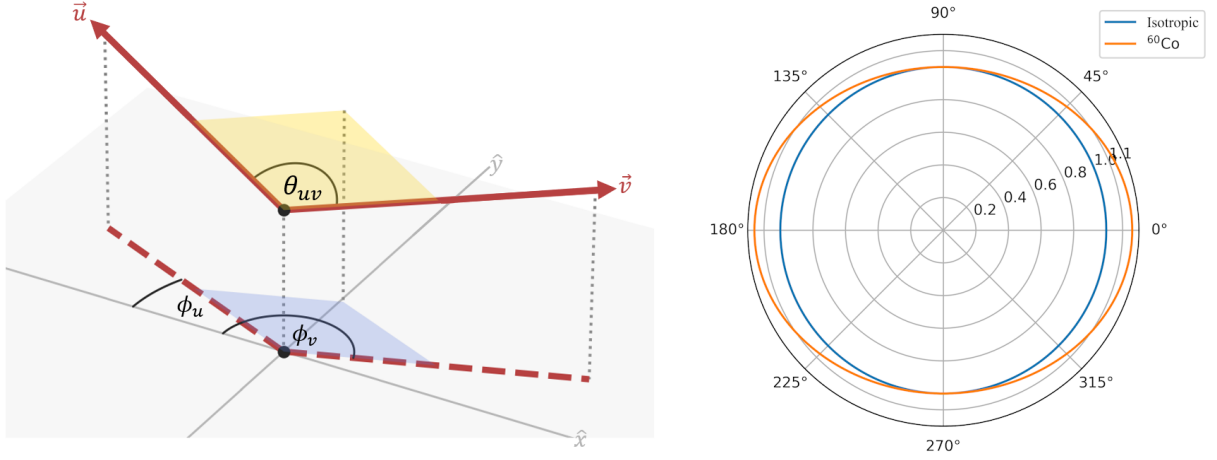


Figure 1. (Left) The cascade geometry for photon pairs emitted in the $\vec{u}$ and $\vec{v}$ directions, θ_{uv} describes the opening angle between the pair, while ϕ_u and ϕ_v describe projections of each direction onto the xy -plane. (Right) θ_{uv} presented on a polar plot relative to the $-\hat{x}$ -direction with radius proportional to $W(\theta_{uv})$. An isotropic emission (blue circle) is compared to that of the angular correlated primary cascade in ^{60}Co (orange oval).

Experimentally, individual detectors measure single photons. Pairs are reconstructed by operating detectors in time coincidence using a short (ns) coincidence gate. The angular correlation is determined by measurements of successive pairs using detectors placed at different positions in space. While in principle the coincidence counting rate provides sufficient information to resolve the angular correlation, it is useful to observe the singles detection rates (s^{-1}) in each detector and the prompt and delayed coincidence rates (s^{-1}) between detector pairs. For singles count rates, r_i and r_j , as recorded by detectors i and j , prompt P_{ij} and delayed D_{ij} coincidence rates between the ij pair, the measured intensity $S(\theta)$ is determined by the ratio:

$$S(\theta) = \frac{r_i r_j}{(P_{ij} - D_{ij})} W(\theta) \quad [2]$$

Forming the ratio in this manner divides out the individual detector efficiencies and solid angle terms that are shared in common between both of the singles and coincidence measurements, thus removing their otherwise high uncertainty contributions. Normalisation by $W(90^\circ)$ allows for direct comparison to the theoretical prediction.

In a previous study, a proof-of-concept measurement consisting of two detectors connected to a data acquisition system was sufficient to constrain the a_2 coefficient for the primary $\gamma - \gamma$ cascade in ^{60}Co [2]. However, improved angular resolution would be needed to constrain the a_4 coefficient. In this article, we explore increasing the number of detectors to enhance these measurements. Increasing detector multiplicity improves the angular resolution, angular sampling, and measurement statistics, but also greatly increases the complexity of the measurement and analysis. A consistent computational framework has been developed to model arbitrary detector geometries, optimise experimental configurations, and ultimately extract the angular correlation coefficients from measured singles and coincidence data. The computational framework is shared between the experimental hardware and a deterministic and stochastic simulation suite, enabling combined analysis of experimental and simulation results.

2 Method

We have developed a generalised toolchain enabling interpretation and analysis of any detector geometry used for the measurement of angular correlations. Fig. 2 shows the relationships between the different hardware and software components. Validation begins with the implementation of a common instrumentation module which includes the geometry engine defining individual detector positions (vector) and orientations (tensor), alongside detector characterisation parameters including efficiencies, position-, time-, and spatial- resolutions, and detector calibration parameters. The model is global to the experimental hardware and software tools, ensuring that all analyses methods employ an identical detector description. The instrument model provides a common description of the detector geometry, calibration, and readout, ensuring consistency between deterministic calculations, stochastic modelling, FLUKA simulations, and experimental event extraction.

An example detector array has been built around a series of detector blocks and associated readout electronics. Each detector block consists of an 8×8 segmented BGO (bismuth germinate) scintillator crystal, optically connected to 4 photomultiplier tubes (PMTs) [3]. Two classes of segmented detector are used: 11 high spatial resolution detector blocks (pixel size $40 \times 30 \times 30$ mm) are arranged on an arc opposing a single, larger detector block (pixel size $50 \times 50 \times 30$ mm), giving a total of 768 detector crystals. A point-like source is located at the focal point. The segmented detectors offer a high angular resolution due to the small pixel size and high angular sampling due to the large number of possible detector pairs. The high intrinsic efficiency of 55-66% results in correspondingly high counting statistics. While the energy resolution is relatively poor, the time resolution of ± 2 ns is sufficient to identify both photons from a single cascade. Note that while the geometry chosen for the internal validation of the toolchain is considered plausible for future angular correlation measurements, it has not been optimised for this purpose.

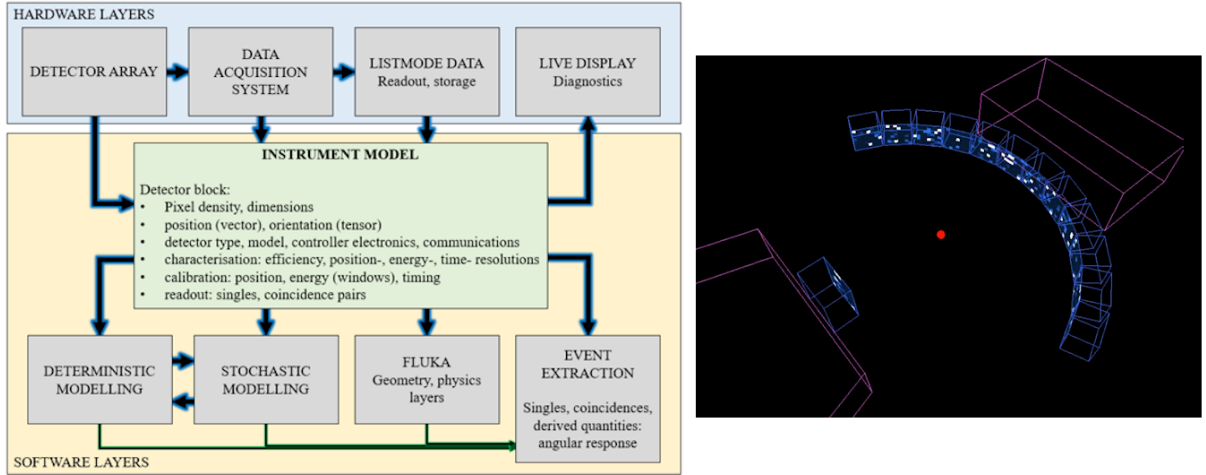


Figure 2. (Left) Framework illustrating the flow of information from the detector array through data acquisition and the instrument model to the software analysis chain. (Right) The software description of the physical detector array, with Monte Carlo generated singles events.

In validating the framework, including the experimental and simulation data acquisition, event selection, and angular correlation analysis, we consider the geometric acceptance of singles and coincidence events at three levels of physical detail. A deterministic calculation represents the detector system from the perspective of the acquired listmode data, in which each detector crystal is represented by its centroid position. A Monte Carlo extension accounts for finite source and detector volumes by repeated sampling of emission and interaction positions within the system geometry. In these codes, for each source position, the source-to-detector vectors corresponding to every possible coincidence pair are determined and compared to a full Monte Carlo model using the commercial FLUKA code [4] capable of describing geometry, materials, and transport effects including attenuation and multiple scattering. Previous studies identified that the angular correlation of nuclear cascades is not inherently encoded into FLUKA [2, 5]. Thus, a custom source routine was created to support the simulation of, and interpretation of, the measured angular correlation. From the theoretically described Probability Distribution Function (PDF) for $W(\theta)$ a Cumulative Distribution Function (CDF) was obtained. The CDF forms the basis of the source routine with random sampling of tabulated values with linear interpolation [5]. Emission of a pair of correlated photons is regarded as the FLUKA primary. The direction vector of the first photon is isotropically sampled, with the relative direction of the second photon sampled from the user defined CDF to resolve a correlated pair. In addition to the source routine, we have developed a custom scoring routine that allows for the correlated scoring of both singles and coincidence events. A listmode file is produced per cycle containing the pixel boundary crossing positions and energies for photon pairs on an event-by-event basis. See [2, 5] for discussion and validation simulations for unmodified (uniform) and modified (^{60}Co) source cases.

3 Results

The deterministic and Monte Carlo calculations represent the detector response at the data acquisition, physical instrument, and combined detector-plus-simulation levels respectively. Regardless of origin, two angular observables are accumulated as histograms: the opening angle between each vector pair (θ_{uv}), and the angles (ϕ_u, ϕ_v) between their projections as described by Fig 1. All histograms are presented with a bin width

of 1° . These data are sufficient to separate the corresponding angular distributions from the system geometry and detector calibration, photon transport effects, and therefore resolve the underlying angular emission process producing the conventional polar plots. Thus, a complete understanding of the detector and data acquisition response can be combined with emission simulation models leading to geometric and physical correction factors and reconstruction of the angular correlation observable. These studies establish the framework required for detector calibration and future experimental measurements providing the foundation for characterising coincidence performance and validating the spectrometer.

The detector array described above (shown in Fig. 3) was written into the shared geometry definition enclosed in the instrument model of Fig. 2. From the definition, the deterministic, deterministic with Monte Carlo extension, and FLUKA models were used to estimate the system responses for an isotropic source placed at the origin. In the FLUKA model photon interaction physics was turned off such that only the geometric response was simulated, with the expectation that results should be equivalent to the deterministic-based geometric models. Fig. 3 shows the detector geometry (left) and the resulting singles detection histograms for all codes (right), where the frequency distribution of events is plotted as a function of the angle each photon makes with the x -axis as defined in Fig. 1 (left). Note that there are two entries to the angle histogram per photon pair (one for each photon).

Due to the placement of the detector blocks, there is no angular acceptance at certain angles, while other angles have a higher probability of being measured than others. It is necessary to determine this angular acceptance before analysing the acquired listmode data to form the measured angular correlation. As the deterministic models simply connect the source-detection points (either as centroids as in the DAQ representation, or as points within the distributed volume) they do not model the individual solid angles subtended by the source onto the detectors. The surface of the detector blocks are flat such that the pixels at the same azimuthal angle do not subtend the same solid angle. Consequently, pixels with the same azimuthal angles record different detected count rates. As the FLUKA model is a full transport model it therefore includes the solid angle terms discarded above. Therefore, the individual singles count rates can be determined precisely, forming the chevron shaped features per detector block in Fig. 3. which arise because of the small difference in radial distance when placing a planar detector onto a sphere. The histograms in Fig. 3. represent the corresponding results for the deterministic codes, being in remarkable agreement with the FLUKA count rate data. Features present in all distributions, such as the reduced intensity at 0° and low intensity gaps between the detector blocks, are replicated by all models.

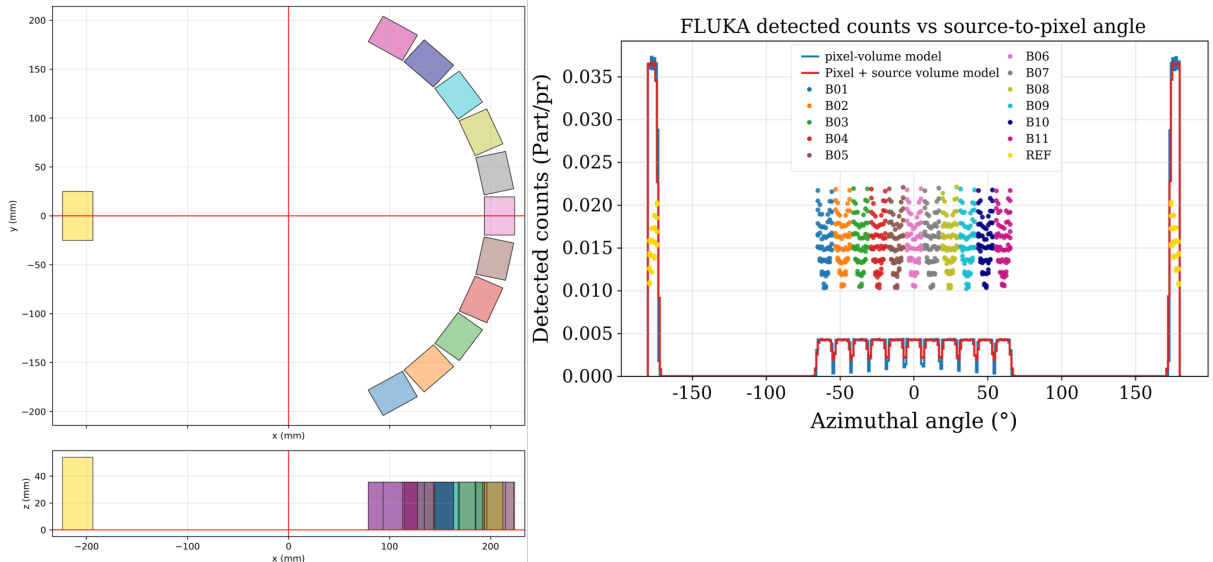


Figure 3. (left) Cartesian projections of the block detector positions, each 8×8 pixel block represented by the coloured boxes drawn to scale. (right) The coloured data points are FLUKA-detected counts per pixel, per primary pair, with each colour corresponding to the complementary detector block on the left. The histograms show the deterministic geometry model results. Both points and histograms are formed from the singles events projected onto the azimuthal angle (θ_i) between the source and detector.

The data shown in Fig. 3. demonstrate that the three models agree, and that the deterministic models are sufficient proxies for the formal calculation. Thus, these results demonstrate that it is possible to obtain angular measurements from appropriate analysis of the acquired listmode data. Fig. 4. illustrates the process: histograms of the opening angle θ_{uv} between photon pairs are shown for all three models (left), the corresponding angular distributions are shown as polar plots (middle), the polar plot using the FLUKA model to correct for the geometric acceptance of the deterministic model (right). The discrete nature of the point-to-point deterministic model is due to the centroid description of the detector pixels, rather than statistical uncertainty. Incorporating the volumetric detector and source descriptions rapidly smooths the angular response histogram, bringing the deterministic calculation into close agreement with the FLUKA simulation.

These data demonstrate that it is possible to take the acquired listmode singles data from an arbitrary detector geometry, form pairs through coincidence processing, and determine their angular correlations. The measured distribution (calculated from listmode acquisition) can be corrected for geometric acceptance (from the equivalent simulation model) to resolve the angular distribution of the emission physics, as illustrated in the figure for an isotropic pairing.

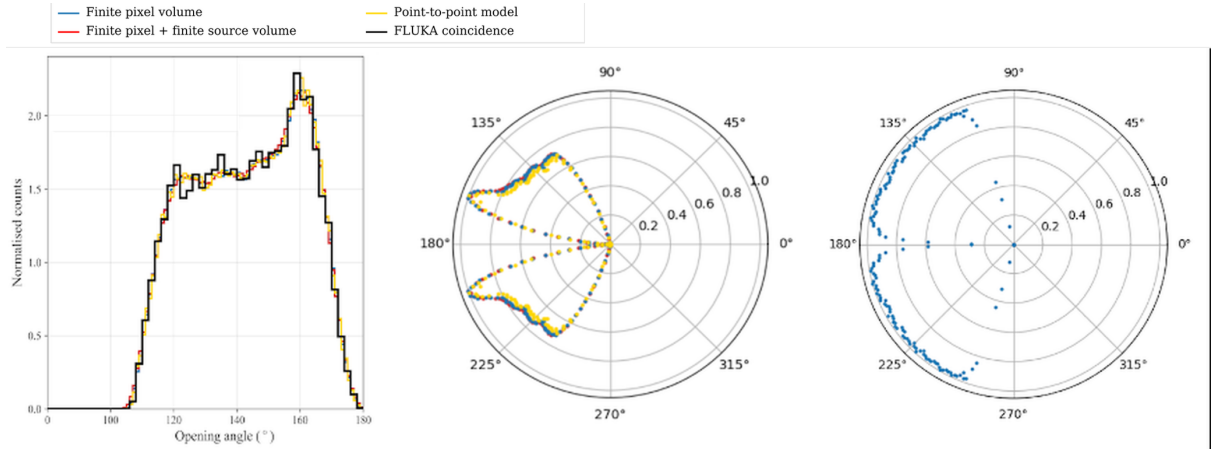


Figure 4. (Left) Histograms of the opening angle (θ_{uv}) acceptance for coincidence pairs in the 3 models. (Middle) The corresponding polar plot equivalent to the measured distribution for an isotropic source. (Right) The polar plot corrected for the detector response, illustrating that the isotropic $W(\theta)$ is recoverable from the analysis.

The results confirm that the geometry has been implemented consistently and provides confidence that the simulation accurately reproduces the experimental configuration. Future discrepancies can be attributed to the underlying transport physics or angular-correlation effects, rather than differences in the geometric model. The detector array geometry used in these studies has been reproduced in the laboratory, where the two classes of segmented detector are arranged as shown in Figs. 2 and 3. The 12 detector blocks have been successfully integrated with a high-speed DAQ, where digitised singles events are transmitted to the data acquisition system, timestamped, and written to disk storage in listmode format at a deadtime limited rate of 2 MHz per block. The detector system is undergoing calibration and characterisation in terms of energy, efficiency, and timing, to establish a reliable experimental baseline.

4 Conclusion

A multidetector spectrometer was assembled to improve the angular resolution of angular correlation measurements in $\gamma - \gamma$ cascade systems. In support of the experimental analysis, a suite of deterministic and Monte Carlo based software modules sharing a common instrument model together with the data acquisition system were developed. The detector models and simulation are shown to be in close agreement for both singles emissions and coincidence pairs. The use of the modelled geometric response in correcting measured data has been illustrated for an isotropic source, demonstrating that the arbitrary detector array and associated software toolchain are capable of measuring angular distributions in γ -pair cascades.

Future work will complete the commissioning and characterisation of the pixelated spectrometer. Benchmark verification results will be produced and analysed for the example case of ^{60}Co with the ambition to resolve the

a_4 term to high precision. From these measurements, investigation of higher order coincidences, and applications of the approach will be explored.

Acknowledgements

This work was supported in part by the UCT Science Faculty PPI fund, as well as the National Research Foundation of South Africa (Ref CSRP250306301626).

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