A pQCD Baseline for Parton and Hadron Production Spectra

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SPES BONA

NRF National Research Foundation

iThemba LABS Laboratory for Accelerator Based Sciences

CERN

Abstract

- We reproduced (almost consistently) LO pQCD differential cross-sections for charged meson ($\pi^{\pm}, \mathrm{K}^{\pm}, \mathrm{p}, \bar{\mathrm{p}}$) production
- Used contemporary parton distribution functions (PDFs) and fragmentation functions.
- Compared with experimental data of p+p at $\sqrt{s} = 630,\ 2000,\ 5360$ GeV.

Introduction

In hard pQCD, processes with final-state hadrons are described in terms of

- Perturbative hard-scattering cross-sections $\mathrm{d}\hat{\sigma}^{ij\to kl}$
- Non-perturbative structure functions $f_i^A(x, Q^2)$,$D_{f\to h}(z, \mu_F^2)$

Schematically,

$$\frac{\mathrm{d}\sigma^{AB\to h+X}}{\mathrm{d}\Phi} = \sum_{i,j,k} D_{k\to h} \otimes f_i^A \otimes f_j^B \otimes \frac{\mathrm{d}\sigma^{ij\to k+X}}{\mathrm{d}\Phi} \tag{1}$$

Collins, Soper, & Sterman (1989), arXiv:hep-ph/0409313

Structure Functions

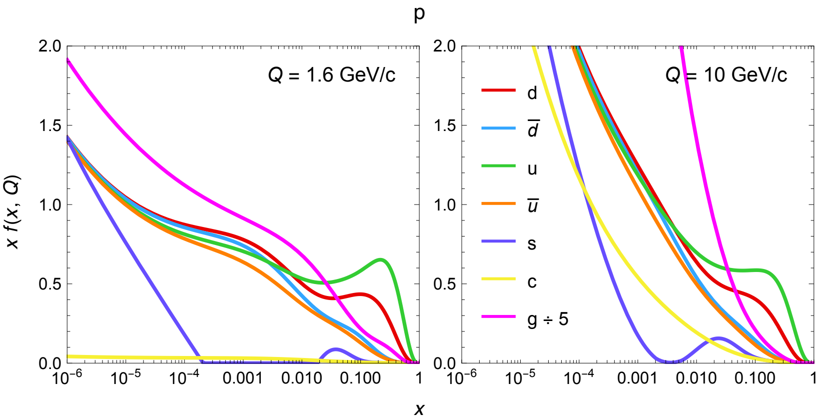
DGLAP renormalisation group equation:

$$\frac{\partial f_i^A(x,Q^2)}{\partial \ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\sum_j \int_x^1 \frac{dx'}{x'} P_{ij}(x') f_j^A(x/x', Q^2) \qquad (2)$$

$$\frac{\partial D_{i\to h}(z,\mu_F^2)}{\partial \ln \mu^2} = \frac{\alpha_s(\mu_F^2)}{2\pi}\sum_j \int_{z'}^1 \frac{dz'}{z} P_{ji}(z') D_{j\to h}(z/z', \mu_F^2) \qquad (3)$$

Parton Distribution Functions

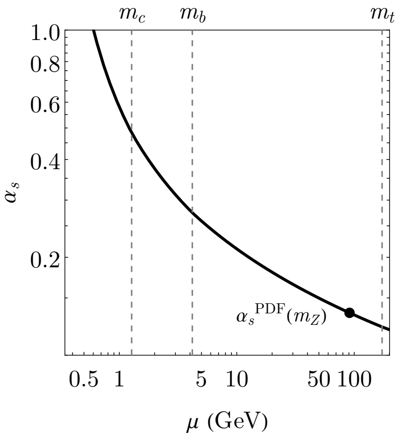
- We used the CT18LO global fit.



Yan, Hou, Nadolsky, & Yuan (2023), arXiv:2205.00137 [hep-ph]

Strong Coupling Running

$$\beta(\alpha_s) = \frac{\mathrm{d}\alpha_s}{\mathrm{d}\ln\mu} = -\beta_0 \frac{\alpha_s^2}{2\pi} - \beta_1 \frac{\alpha_s^3}{(2\pi)^2} + \mathcal{O}(\alpha_s^4) \qquad (4)$$



$$\alpha_s^{(0)}(\mu) = \frac{2\pi}{\beta_0(N_f)\ln\left(\frac{\mu}{\Lambda_{\mathrm{QCD}}(N_f)}\right)},$$

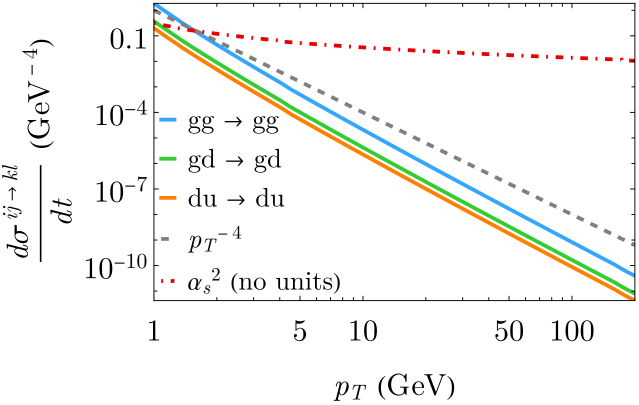
$$\beta_0 = \frac{11C_A - 4T_F N_f}{6} \qquad (5)$$

Callan (1970), Phys. Rev. D 2, 1541

Parton Interaction Spectra

LO interaction cross-section of a hard 2-to-2 QCD vertex:

$$\frac{\mathrm{d}\hat{\sigma}^{ij\to kl}}{\mathrm{d}\hat{t}}(\hat{s},\hat{t},\hat{u}) = \frac{\pi\alpha_s^2}{\hat{s}^2}\left|\left\langle\mathcal{M}^{ij\to kl}(\hat{s},\hat{t},\hat{u})\right\rangle\right|^2 \quad (6)$$



Combridge, Kripfganz, & Ranft (1977)

Parton Production Spectra

- Parton pairs (k, l) produced back-to-back in the transverse plane have the LO cross-section

$$\frac{\mathrm{d}\sigma^{AB\to kl}}{\mathrm{d}p_T^2\,\mathrm{d}y_1\,\mathrm{d}y_2} = \sum_{i,j} x_1 f_i^A(x_1, Q^2)\; x_2 f_j^B(x_2, Q^2)\; \frac{\mathrm{d}\hat{\sigma}^{ij\to kl}}{\mathrm{d}\hat{t}} \qquad (7)$$

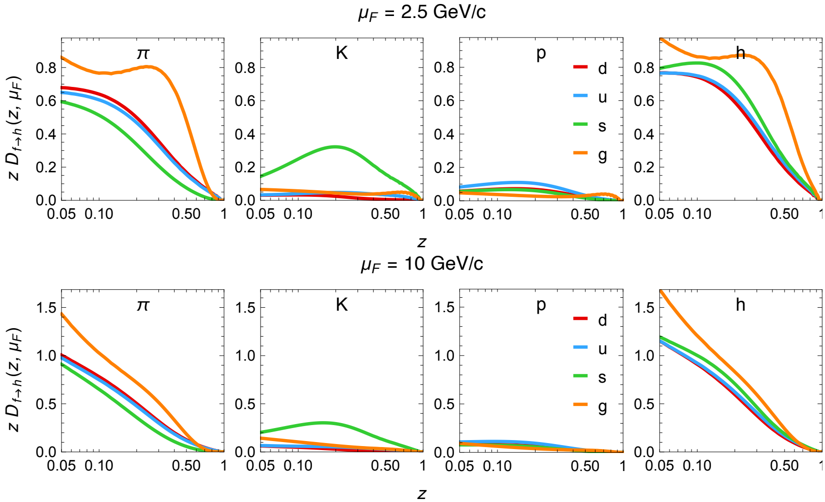
- So the inclusive cross-section to produce parton f is

$$\frac{\mathrm{d}\sigma^{AB\to f+X}}{\mathrm{d}p_T^2\,\mathrm{d}y_f} = \int \mathrm{d}y_1\,\mathrm{d}y_2 \sum_{\langle k,l\rangle} \frac{\mathrm{d}\sigma^{AB\to kl}}{\mathrm{d}p_T^2\,\mathrm{d}y_1\,\mathrm{d}y_2}$$
$$\times\ \left[\delta_{kf}\delta(y_f - y_1) + \delta_{lf}\delta(y_f - y_2)\right] \frac{1}{1+\delta_{kl}} \qquad (8)$$

Eskola & Honkanen (2003), arXiv:hep-ph/0205048

Fragmentation Functions

- We used the DSS dataset, containing $\pi^{\pm}$, $\mathrm{K}^{\pm}$, $\mathrm{p}(\bar{\mathrm{p}})$, $\mathrm{h}^{\pm}$ fragmentation functions from NLO combined analyses.



de Florian, Sassot, & Stratmann (2007), arXiv:hep-ph/0703242

Hadron Production Spectra

The inclusive cross-section to produce hadron h is

$$\frac{\mathrm{d}\sigma^{AB\to h+X}}{\mathrm{d}q_T^2\,\mathrm{d}y} = K(\sqrt{s})\,J(m_T,y)\sum_f\int\frac{\mathrm{d}z}{z^2}D_{f\to h}(z,\mu_F^2)\left.\frac{\mathrm{d}\sigma^{AB\to f+X}}{\mathrm{d}p_T^2\,\mathrm{d}y_f}\right|_{p_T^2,y_f} \quad (9)$$

- $K(\sqrt{s})$ is the beyond-LO factor
- The parton cross-section is evaluated at

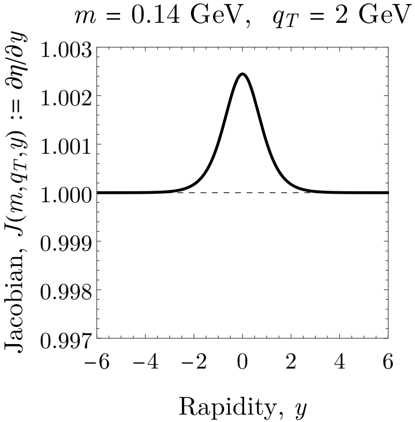
$$p_T = \frac{q_T}{z}J(m_T,y)\;, \qquad y_f = \operatorname{arcsinh}\left(\frac{m_T}{q_T}\sinh\,y\right)$$

- The integral bounds are

$$\frac{2m_T}{\sqrt{s}}\cosh\,y \;\le\; z \;\le\; \min\left(1,\;\frac{q_T}{p_0}J(m_T,\,y)\right)$$

Eskola & Honkanen (2003), arXiv:hep-ph/0205048

$$\eta = \mathrm{arcsinh}(q_z/q_T) \ , \quad y = \mathrm{arcsinh}(q_z/\sqrt{q_T^2 + m^2}) \tag{10}$$



Hadron Production Spectra

The inclusive cross-section to produce hadron h is approximately

$$\left.\frac{\mathrm{d}\sigma^{AB\to h+X}}{\mathrm{d}q_T^2\,\mathrm{d}y}\right|_{q_T\gg m} \approx K(\sqrt{s})\sum_f \int \frac{\mathrm{d}z}{z^2} D_{f\to h}(z,\mu_F^2)\left.\frac{\mathrm{d}\sigma^{AB\to f+X}}{\mathrm{d}p_T^2\,\mathrm{d}y_f}\right|_{p_T^2,y_f} \tag{11}$$

- $K(\sqrt{s})$ is the beyond-LO factor
- The parton cross-section is evaluated at

$$p_T = \frac{q_T}{z}\,, \qquad y_f = \operatorname{arcsinh}\left(\frac{m_T}{q_T}\sinh\eta\right)$$

- The integral bounds are

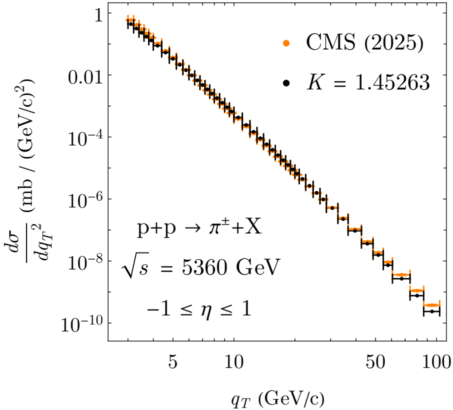
$$\frac{2m_T}{\sqrt{s}}\cosh\eta \;\leq\; z \;\leq\; \min\left(1,\; \frac{q_T}{p_0}\right)$$

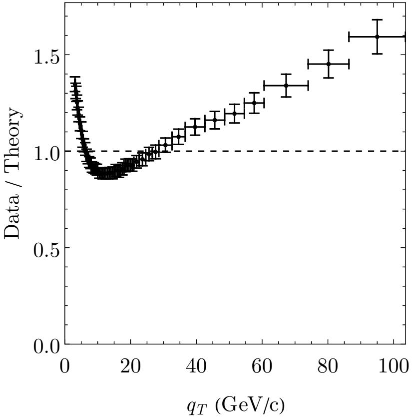
```
double precision :: sqrt_s = 5360.d0  ! CME (in units of pt)

character(*) :: h_strs(3) = ['p', 'He4', 'Pb208']

character(*) :: pdf_files_lo(3) = [
    'CT18LO.LHgrid', &
    'EPPS21nlo_CT18Anlo_He4.LHgrid', &
    'EPPS21nlo_CT18Anlo_Pb208.LHgrid']
! Alternative LO proton-PDF files:
!   'HERAPDF20_LO_EIG.LHgrid'
!   'MMHT2014lo68cl.LHgrid'
!   'NNPDF40_lo_as_01180.LHgrid'

character(*) :: pdf_files_nlo(3) = ['CT18NLO.LHgrid', '', '']
! Alternative NLO proton-PDF files:
!   'CT18ANLO.LHgrid'
!   'MSHT20nlo_as118.LHgrid'
!   'NNPDF40_nlo_as_0118.LHgrid'
```

```fortran
! Transverse momentum points in GeV
double precision :: pt_arr(*) = [2.d0, 3.d0, 4.d0, 5.d0, 6.d0, &
7.d0, 8.d0, 9.d0, 10.d0, 12.5d0, 15.d0, 17.5d0, 20.d0, 30.d0, &
40.d0, 50.d0, 60.d0, 70.d0, 80.d0, 90.d0, 100.d0]

! Rapidity range of produced hadron
double precision :: y_min = -1.d0
double precision :: y_max = 1.d0

! Outgoing (produced) hadron
integer :: specie = 4          ! 1: pi,  2: K,  3: p,  4: h
integer :: charge = 0          ! -1: -,  0: avg,  1: +
integer :: set = 0             ! 0: best fit,  +-i: hessian vars
character(2) :: conf = '90'    ! '90': 90%,  '68': 68%
character(2) :: year = '07'    ! '07': 2007,  '21': 2021
```

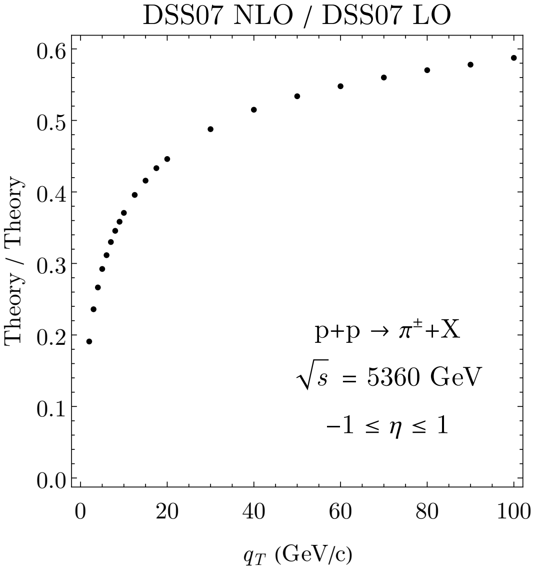
```fortran
! perturbative order
integer :: order = 1  ! 1: lo,  2: nlo

! Factorization scales squared (in units of pt**2)
double precision :: scale_pdf = 1.d0
double precision :: scale_alpha = 1.d0
double precision :: scale_ff = 1.d0

! Integration options
logical :: integrate_pt = .false.
logical :: integrate_eta = .true.
double precision :: eta_min = y_min
double precision :: eta_max = y_max

! Vegas integration arguments
double precision :: vegas_tolerance = 1.d-6
integer :: vegas_ncalls = 100000
integer :: vegas_iter_max = 5
```

Hadron Production Spectra





Hadron Production Spectra



Thank you!

Questions

Cross section phase space

$$\mathrm{d}\hat{\sigma}^{ij\to kl} = \frac{1}{2\hat{s}}(4\pi\alpha_s)^2|\langle\mathcal{M}^{ij\to kl}\rangle|^2\,\mathrm{d}\Phi \tag{12}$$

$$\begin{aligned}\mathrm{d}\Phi &= (2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4)\frac{1}{2E_3}\frac{\mathrm{d}^3p_3}{(2\pi)^3}\frac{1}{2E_4}\frac{\mathrm{d}^3p_4}{(2\pi)^3}\\ &= \begin{cases}\dfrac{1}{8\pi\hat{s}}\,\mathrm{d}\hat{t}\\ \dfrac{1}{16\pi^2}\,\mathrm{d}p_T^2\,\mathrm{d}y_1\,\mathrm{d}y_2\end{cases}\end{aligned} \tag{13}$$

$$\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}p_T^2\,\mathrm{d}y_1\,\mathrm{d}y_2} = \frac{1}{2\pi}\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\hat{t}} \tag{14}$$