Saved by the Bell

Geoff Beck

geoff.m.beck@proton.me

07 July SAIP 2026

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

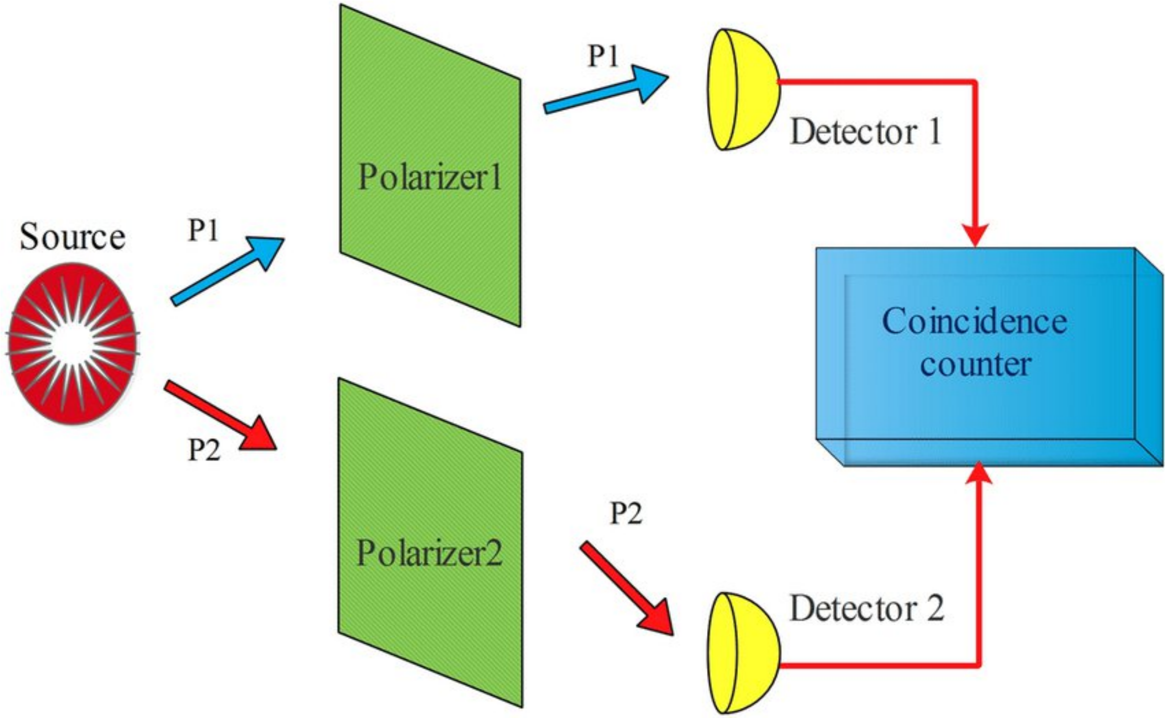
SOUTH AFRICAN INSTITUTE OF PHYSICS

Based on arXiv: 2605.29589

Bell’s theorem

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$$

- We start with two experimenters A and B
- Each receives a spin drawn from an entangled state
- Each measures their spin on some arbitrary axis
- They compare notes to determine the correlation between their results



The result

- The resulting correlation is given as

$$C_{AB}(\varphi, \theta) = -\cos(2(\varphi - \theta))$$

- The conditional probabilities take the form

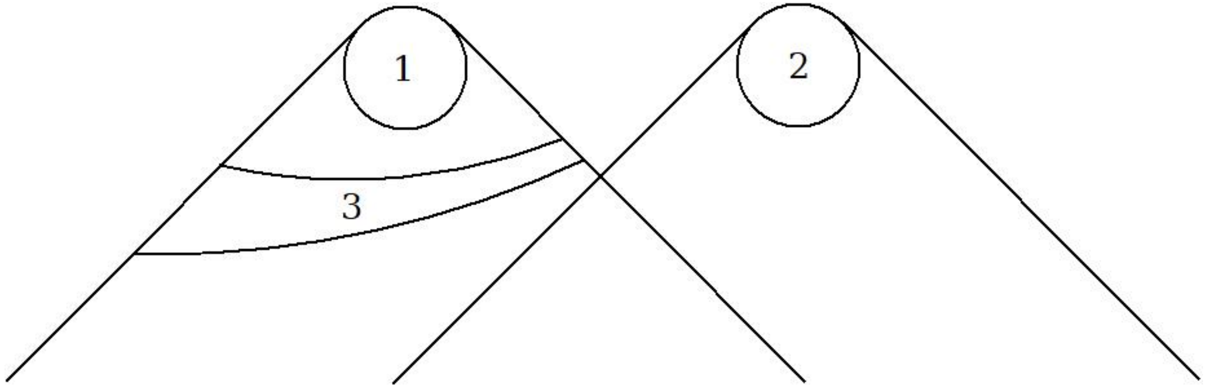
$$P\left(A_{\uparrow,\theta} | B_{\downarrow,\varphi}, \psi\right) = \cos^2(\varphi - \theta) \qquad P\left(A_{\uparrow,\theta} | B_{\uparrow,\varphi}, \psi\right) = \sin^2(\varphi - \theta)$$

- Which is taken to indicate a possible causal connection between measurements as

$$P\left(A_{\uparrow,\theta} | B_{\downarrow,\varphi}, \psi\right) \neq P(A_{\uparrow,\theta} | \psi)$$

- However, the measurements can be done at spacelike separation
- This indicates a non-local connection between outcomes

Bell's assumptions



1. Spacetime is Lorentzian
2. Causation is consistent with 1
3. Probability completeness

$$P(Y) = \sum_X P(X)P(Y|X)$$

4. Common cause screening

$$P(A, B|X) > P(A|X)P(B|X)$$

→ A causes B
B causes A
Something in X causes both

5. Probability factorisation

$$P(A|B, X) = P(A|X)$$

→ If 4 is true, X can contain a common cause

Myrvold et al, Stanford Encyclopedia of Philosophy

Which assumption fails?

- In quantum mechanics it is probability factorisation

$$P\Big(A_{\uparrow,\theta}|B_{\downarrow,\varphi},\psi\Big) = \cos^2(\varphi-\theta) \qquad P\Big(A_{\uparrow,\theta}|B_{\downarrow,\varphi},\psi\Big) \neq P(A_{\uparrow,\theta}|\psi)$$

- However, this can easily fail in classical cases

A machine deals out 2 coins showing opposite faces.

$$P(A=1|B=0) = 1 = P(B=1|A=0)$$

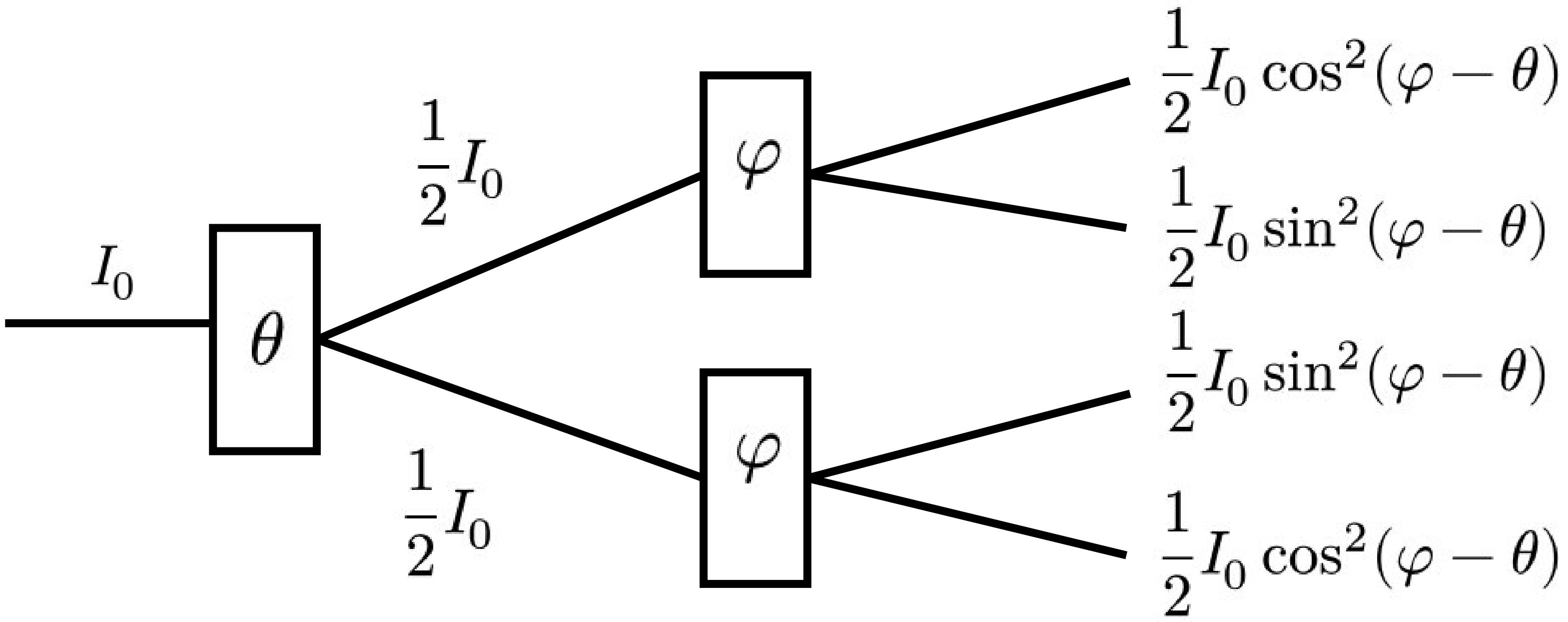
$$P(B=0) = \frac{1}{2} = P(A=0)$$

$$P(A=1, B=0) = \frac{1}{2} > P(A=1)P(B=0)$$

Implies causal link between A and B, despite existing common cause

A classical polarisation test

- We start with an unpolarised state
- Pass through two layers of polarising beam splitter



- The correlation function matches Bell up to a sign
- Is this a coincidence?

$$C(\varphi,\theta)=\cos(2(\varphi-\theta))$$

Coincidence or not?

- Our Bell state particles are highly correlated
- If we measure both on the same axis we get perfect matching/anti-matching

$$P\Big(A_{\uparrow,\theta}|B_{\downarrow,\theta},\psi\Big)=1$$

- This means our conditional probabilities obey

$$P\Big(A_{\uparrow,\theta}|B_{\downarrow,\varphi},\psi\Big)=P\Big(A_{\uparrow,\theta}|A_{\uparrow,\varphi},\psi\Big)$$

- What is this probability?
- It is that of successive measurement results on spin A alone
- Interestingly,

$$\mathrm{Tr}\Big(\hat{\rho}\hat{\sigma}_{\theta}\hat{\sigma}_{\varphi}\Big)=\cos(2(\varphi-\theta))=\mathrm{Tr}\Big(\hat{\rho}\hat{\sigma}_{\varphi}\hat{\sigma}_{\theta}\Big)$$

- **Our successive and entangled experiments are two versions of the same measurement**

What does it imply?

- There is nothing non-local/entangled about our successive experiment
- The results are just from the physics of filtering spins

- **Could this also be true for the entangled mirror case?**

- It is odd that the correlation is the same with different explanations
  - Entangled case: non-locality
  - Successive case: local physics
- Could we use the matching correlation to explain the physics?

Directional asymmetry

- The entangled case can't explain the successive one

- However, the same local physics is present in both cases
- Says that measuring A tells us the B value on the same axis
- This is a counterfactual
- The two experiments are counterfactual mirrors
- Quantum mechanics is "counterfactually definite"

$$P\Big(A_{\uparrow,\theta}|B_{\downarrow,\varphi},\psi\Big) = P\Big(A_{\uparrow,\theta}|A_{\uparrow,\varphi},\psi\Big)$$

Measuring B tells me the unmeasured response of A!

Counterfactuals and non-locality

- The entanglement allows us to counterfactually measure B via A
- This is why the correlation is the same
- Entanglement doesn't add extra physics
- It allows us to test the same physics differently

- This explains why our "non-locality" violates the spirit of relativity but not the letter
- It isn't actually non-local

If there’s no non-locality, why don’t local hidden variable models work?

The LHV problem

- Let’s take a look at how these models are often framed

$$S_\theta = \mathrm{sgn}\left(\vec{\theta} \cdot \vec{\lambda}\right)$$

- This yields discrete results.
- It is still classical
- It is assumed there is a vector description of spin

Bell, 1965

The alternative?

- Spins have the same “projection” on every axis
- This cannot be encompassed by a vector
- Quantisation of the spin is what prevents this

- It’s possible that what hidden variable models are missing is quantisation

- Of course, it’s not clear how we could incorporate this mathematically!

A Fine solution

- Fine's theorem says that the non-existence of

$$P(s_1, s_2, s_3)$$

- Is both necessary and sufficient for Bell violation
- This makes no sense in the context of Bell experiments!
- These deal in two-spin correlations, not three!

- In a successive scenario it just means an extra layer of filters
- Open question: does the third layer requirement reveal any physics?

Fine, 1982
Halliwell, 2014 arXiv: 1403.7136

Conclusions

- There is no coincidence in the successive-entangled matching
- They are counterfactual equivalents
- This requires a realist belief in premeasurement values
- Bell experiments are local in this view
- Local hidden variables still fail
- Just due to not taking quantisation seriously?

- Bell says that no local, **classical,** hidden variable model can reproduce QM